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Elastic Moduli and Anisotropy of Dunite to 10 Kilobars

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Longitudinal and transverse wave velocities are reported for several directions at pressures to 10 kb in two samples of dunite from the Twin Sisters peaks, Washington. For both dunites, elastic-wave propagation is controlled to a large extent by olivine fabric. Dunite A, which has a strong concentration of olivine *a* crystallographic axes and girdles of *b* and *c* axes, is uniaxial in elastic properties. The longitudinal wave velocity at 10 kb for propagation parallel to the olivine *a* axes maximum is 8.76 km/sec. For propagation normal to the *a* axes maximum, longitudinal wave velocities are low ($V_p = 7.98$ km/sec at 10 kb) and two transverse waves ($V_s = 4.41$ and 4.69 km/sec at 10 kb) are clearly transmitted through the rock. Dunite B, with strong concentrations of all three olivine crystallographic axes, is similar in elastic properties to orthorhombic crystals with a high longitudinal wave velocity (9.15 km/sec at 10 kb) along the olivine *a* axes maximum and a low longitudinal wave velocity (7.83 km/sec at 10 kb) along the olivine *b* axes concentration. Elastic stiffnesses and compliances were computed from the velocities, and the physical properties of isotropic aggregates of the two dunites were calculated using the Voigt and Reuss averaging techniques. Primarily because of the presence of accessory minerals in the dunites, the Voigt and Reuss velocities are lower than values computed from olivine single-crystal data. The high pressure gradient ($\partial V_p/\partial P =$ 17.0 km sec^{-1} mb^{-1}) observed for the longitudinal velocity of dunite B at 8 kb is interpreted as being due to the effect of grain boundary cracks.

Several papers [*Hess*, 1964; *Raitt et al.*, 1969; *Meyer et al.*, 1969; *Keen and Tramontini*, 1970] have presented evidence from seismic refraction studies that parts of the oceanic upper mantle are anisotropic to compressional wave propagation. If this anisotropy is the result of preferred mineral orientation, seismic refraction data may eventually provide important information on mineral fabric in the upper mantle, which in turn may lead to a better understanding of flow mechanisms and flow directions associated with plate motion. Since olivine is generally believed to be a major constituent of the upper mantle, it is of importance to understand the relationship between seismic-wave propagation and fabric in dunite.

Very little experimental data are available on seismic anisotropy of dunites at high pressures. *Birch* [1960, 1961] was the first to find that longitudinal wave velocities vary significantly with propagation direction in dunites. In Birch's study, measurements were made in three mutually perpendicular directions. Although fabric diagrams were not given, *Birch* [1961] found that the fast direction in a specimen of Twin Sisters dunite was parallel to a strong concentration of olivine *a* axes. *Christensen* [1966*b*] reported longitudinal wave velocities in three directions for a second sample of Twin Sisters dunite and in seven directions for a dunite from Addie, North Carolina. Fabric diagrams for both samples demonstrated that longitudinal wave anisotropy at pressures above a few kilobars was clearly related to olivine fabric.

Christensen [1966*a*] found that transverse wave velocities in metamorphic rocks vary with propagation direction and displacement direction. Although transverse wave velocities have been measured at pressures to 10 kb for different propagation directions in dunite [*Simmons*, 1964; *Christensen*, 1966*b*], the relationships between transverse wave velocities and olivine orientation for different displacement directions along a given propagation direction have not yet been investigated.

In addition to the experimental studies of anisotropic rock elasticity, several papers have attempted to relate preferred mineral orientation in rocks and the elasticity of their constituent minerals to a predicted pattern of rock

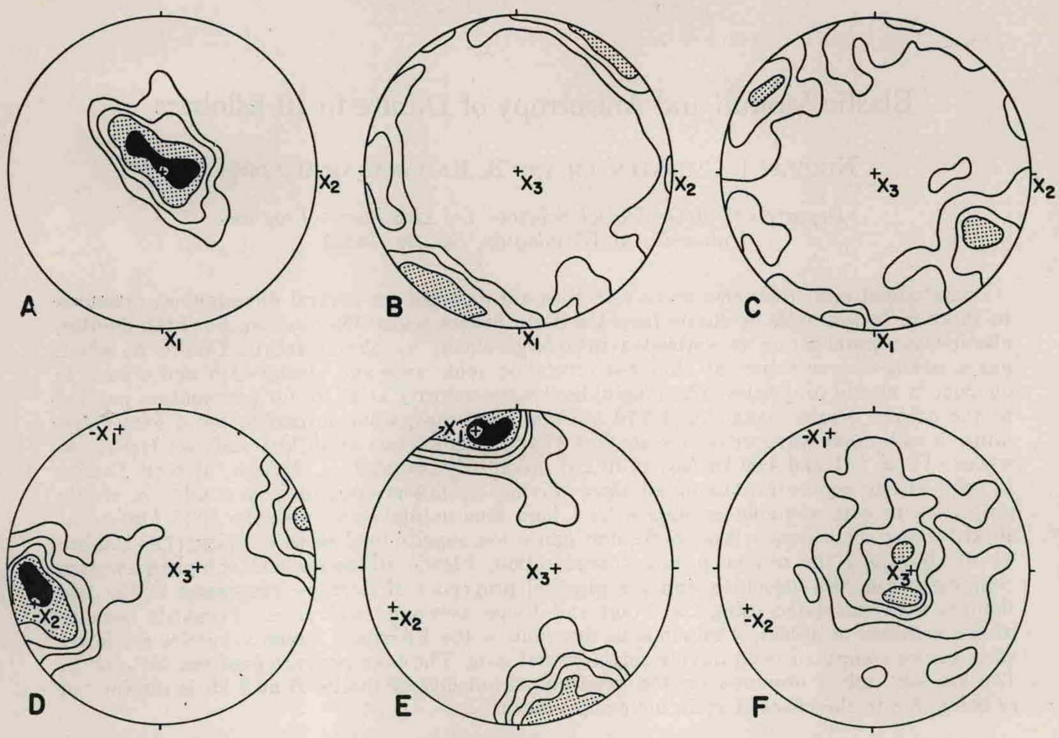

Fig. 1. Orientation diagrams for 200 grains of olivine in dunite A (*A–C*) and dunite B (*D–F*): (*A*) *a* axes, contours 10%, 5%, 3%, and 1% per 1% area; (*B*) *b* axes and (*C*) *c* axes, contours 5%, 3%, and 1% per 1% area; (*D*) *b* axes and (*E*) *a* axes, contours 10%, 7%, 5%, 3%, and 1% per 1% area; (*F*) *c* axes, contours 7%, 5%, 3%, and 1% per 1% area.

anisotropy. *Kumazawa* [1964] presented a stimulating discussion of the effect of grain boundaries and preferred mineral orientation on the elastic properties of rocks and proposed theoretical models for the elastic anisotropy of dunites with different olivine orientation patterns. *Christensen and Crosson* [1968] summarized available fabric data of dunites and peridotites and postulated that, depending on the details of their fabric, the elasticity of common olivine-rich rocks is similar to either hexagonal or orthorhombic crystals. *Alexandrov et al.* [1969] have described a systematic technique for studying the complete elastic properties of rocks and have reported elastic moduli for several anisotropic rocks at atmospheric pressure.

We have studied in detail the elastic anisotropy

TABLE 1. Modal Analyses (Percentages by Volume)

Mineral	Dunite A	Dunite B
Olivine	83.1	96.3
Enstatite	2.1	0.9
Serpentine	12.1	0.2
Chromite + Magnetite	2.7	2.6

TABLE 2. Chemical Analyses (Percentages by Weight)

Component	Dunite A	Dunite B
SiO_2	40.8	40.5
Al_2O_3	0.26	0.78
TiO_2	0.25	0.11
FeO	7.80	8.34
Fe_2O_3	0.92	0.53
MgO	49.1	48.5
CaO	0.02	0.13
K_2O	0.01	0.07
Na_2O	0.20	1.30
MnO	0.03	0.15
Cr_2O_3	0.19	0.62
H_2O	1.30	0.16
Total	100.67	101.19

TABLE 3. Elastic Wave Velocities

Core Density, g/cm³	Direction of Wave Propagation	Direction of Wave Displacement	Notation	Velocity Relation	Velocity, km/sec 1.0 kb	2.0 kb	4.0 kb	6.0 kb	8.0 kb	10.0 kb
				Dunite A (x_3 *axis//to symmetry axis*)						
3.260	[001]	[001]	V_{A1}	$\rho V^2 = c_{33}$	8.568	8.622	8.688	8.726	8.750	8.762
		[100]	V_{A2}	$\rho V^2 = c_{44}$	4.507	4.530	4.560	4.578	4.589	4.597
		[010]	V_{A3}	$\rho V^2 = c_{44}$	4.506	4.531	4.565	4.587	4.595	4.598
3.224	[100]	[100]	V_{A4}	$\rho V^2 = c_{11}$	7.854	7.896	7.946	7.972	7.992	8.007
		[010]	V_{A5}	$\rho V^2 = \frac{1}{2}(c_{11} - c_{12})$	4.352	4.381	4.414	4.430	4.440	4.447
		[001]	V_{A6}	$\rho V^2 = c_{44}$	4.572	4.599	4.635	4.660	4.671	4.677
3.248	[010]	[010]	V_{A7}	$\rho V^2 = c_{11}$	7.797	7.839	7.896	7.922	7.945	7.957
		[001]	V_{A8}	$\rho V^2 = c_{44}$	4.597	4.621	4.643	4.657	4.667	4.676
		[100]	V_{A9}	$\rho V^2 = \frac{1}{2}(c_{11} - c_{12})$	4.319	4.341	4.375	4.393	4.400	4.405
3.296	[110	[110]	V_{A10}	$\rho V^2 = c_{11}$	7.769	7.813	7.857	7.884	7.900	7.912
		$[1\bar{1}0]$	V_{A11}	$\rho V^2 = \frac{1}{2}(c_{11} - c_{12})$	4.344	4.360	4.379	4.392	4.402	4.415
		[001]	V_{A12}	$\rho V^2 = c_{44}$	4.577	4.618	4.665	4.683	4.694	4.696
3.269	$[1\bar{1}0]$	$[1\bar{1}0]$	V_{A13}	$\rho V^2 = c_{11}$	7.852	7.905	7.965	8.001	8.026	8.046
		[110]	V_{A14}	$\rho V^2 = \frac{1}{2}(c_{11} - c_{12})$	4.279	4.304	4.335	4.350	4.358	4.362
		[001]	V_{A15}	$\rho V^2 = c_{44}$	4.617	4.647	4.677	4.687	4.689	4.691
3.286	$[0\bar{1}1]$	Quasi-long.	V_{A16}	*a*	8.339	8.383	8.444	8.481	8.501	8.504
3.256	$[\bar{1}01]$	Quasi-long.	V_{A17}	*a*	8.346	8.393	8.464	8.508	8.530	8.536
				Dunite B						
3.300	[100]	[100]	V_{B1}	$\rho V^2 = c_{11}$	8.939	9.002	9.069	9.103	9.128	9.150
		[010]	V_{B2}	$\rho V^2 = c_{66}$	4.667	4.709	4.763	4.786	4.814	4.832
		[001]	V_{B3}	$\rho V^2 = c_{55}$	4.809	4.842	4.885	4.912	4.934	4.946
3.322	[010]	[010]	V_{B4}	$\rho V^2 = c_{22}$	7.664	7.704	7.756	7.788	7.810	7.831
		[001]	V_{B5}	$\rho V^2 = c_{44}$	4.631	4.673	4.733	4.774	4.799	4.814
		[100]	V_{B6}	$\rho V^2 = c_{66}$	4.645	4.692	4.753	4.788	4.806	4.815
3.329	[001]	[001]	V_{B7}	$\rho V^2 = c_{33}$	8.062	8.119	8.182	8.222	8.250	8.272
		[100]	V_{B8}	$\rho V^2 = c_{55}$	4.921	4.931	4.948	4.962	4.971	4.980
		[010]	V_{B9}	$\rho V^2 = c_{44}$	4.634	4.663	4.704	4.727	4.740	4.747
3.315	[011]	Quasi-long.	V_{B10}	*b*	7.915	7.975	8.042	8.092	8.133	8.166
3.327	[101]	Quasi-long.	V_{B11}	*c*	8.438	8.511	8.584	8.627	8.665	8.701
3.308	[110]	Quasi-long.	V_{B12}	*d*	8.450	8.515	8.576	8.613	8.650	8.686

a $$\rho V^2 = \frac{c_{44}}{2} + \frac{c_{33} + c_{11}}{4} + \frac{[(c_{33} - c_{11})^2 + 4(c_{31} + c_{44})^2]^{1/2}}{4}$$

b $$\rho V^2 = \frac{c_{44}}{2} + \frac{c_{22} + c_{33}}{4} + \frac{[(c_{22} - c_{33})^2 + 4(c_{23} + c_{44})^2]^{1/2}}{4}$$

c $$\rho V^2 = \frac{c_{55}}{2} + \frac{c_{33} + c_{11}}{4} + \frac{[(c_{33} - c_{11})^2 + 4(c_{31} + c_{55})^2]^{1/2}}{4}$$

d $$\rho V^2 = \frac{c_{66}}{2} + \frac{c_{11} + c_{22}}{4} + \frac{[(c_{11} - c_{22})^2 + 4(c_{12} + c_{66})^2]^{1/2}}{4}$$

of two dunites with different patterns of olivine orientation. To a first approximation, the elastic properties of the anisotropic dunites are expressed in terms of the elastic constants of hexagonal and orthorhombic materials. The Voigt-Reuss-Hill averaging scheme is used to define the isotropic elastic properties of dunite and the results are compared with simple directional averages that have been applied previously to dunites. The calculated isotropic elastic properties of the dunites are compared with those calculated from olivine single-crystal data.

DATA

Two samples of dunite with different fabrics from the Twin Sisters peaks, Washington, were selected for the study. Olivine fabric diagrams are shown in Figure 1 for the two specimens. The olivine orientations were obtained by standard universal stage techniques from several thin sections cut from different parts of each sample. For both rocks the fabrics appear to be relatively homogeneous.

Cores 2.5 cm in diameter and 5 to 7 cm in length were cut from the samples. Directions for each core were assigned using a conventional Miller indices notation for three orthogonal axes (x_1, x_2, x_3) shown in Figure 1. In sample A, x_3 was taken to be parallel to the olivine *a* axis concentration and x_1 and x_2 were located arbitrarily in the plane normal to x_3. For sample B, x_1, x_2, and x_3 were assigned parallel to the maximum concentrations of olivine *a*, *b*, and *c* axes, respectively.

Average modal analyses from several thin sections are given in Table 1. Chemical analyses of the two samples obtained by standard X-ray fluorescence and atomic absorption techniques are reported in Table 2. The trimmed ends of the cores used for the velocity measurements were crushed for the chemical analyses. Thus the analyses should be fairly representative of the whole rocks.

The technique of velocity measurement is similar to that described by *Birch* [1960]. Barium titanate transducers of 2-MHz frequencies were used to generate and receive the longitudinal waves. AC-cut quartz transducers of 1-MHz frequencies were used for the transverse wave velocity measurements. At pressures above a few kilobars, accuracies are estimated to be ±½% for V_p and ±1% for V_s [*Christensen and Shaw*, 1970]. Pressure was obtained by measuring the change in electrical resistance of a calibrated manganin coil and is accurate to ±1%.

Compressional and shear wave velocities are given in Table 3. Bulk densities were calculated from the weights and dimensions of the cores. The velocities have been corrected for change in length due to compression by using an iterative routine and dynamically determined compressi-

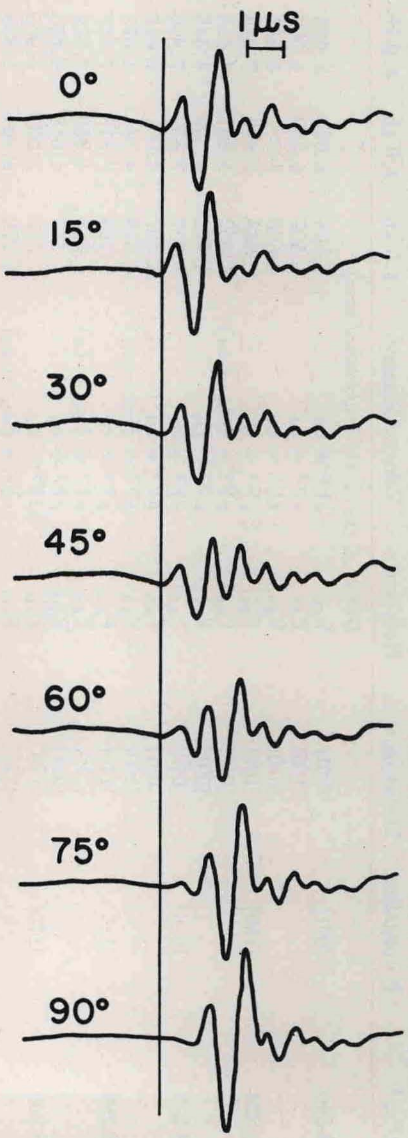

Fig. 2. Oscilloscope traces for shear-wave propagation at 5 kb normal to the olivine *a* axes maximum in dunite A.

bilities. This correction lowers the 10-kb velocities by approximately ½%.

The directions of wave propagation were selected according to the symmetry of olivine fabric for each specimen. Dunite A with a strong maximum of olivine *a* axes was treated as uniaxial, and dunite B with strong concentrations of all three olivine axes was considered orthorhombic in symmetry. The relations between wave velocity, the directions of propagation and displacement, and the elastic constants in materials of hexagonal and orthorhombic symmetry [*Love*, 1944; *Hearmon*, 1961] are given in Table 3, along with the dunite velocities.

DISCUSSION

The fabric of dunite A consists of a strong concentration of olivine *a* axes and girdles of olivine *b* and *c* axes (Figure 1). The symmetry axis of this specimen, which is parallel to the olivine *a* axes concentration, is the direction of fast longitudinal wave velocity. This is in agreement with longitudinal wave velocities measured in single crystals of olivine [*Verma*, 1960; *Graham and Barsch*, 1969; *Kumazawa and Anderson*, 1969] which are fast parallel to the olivine *a* axis.

The several cross checks given in Table 3 clearly demonstrate the uniaxial nature of dunite A. Longitudinal wave velocities for propagation directions normal to the strong olivine *a* axes concentration are similar to one another. Transverse wave velocities for propagation parallel to the olivine *a* axes maximum do not vary significantly with displacement direction. For propagation normal to the olivine *a* axis maximum, two transverse waves with different velocities are propagated through the rock. This is illustrated in Figure 2, where in the upper oscilloscope trace the faster transverse wave is received. Rotation of the transducers through steps of 15° decreases the amplitude of the faster wave until only the slower wave is received.

The fabric diagrams of dunite A show weak maximums of olivine *b* and *c* axes in the x_1-x_2 plane. However, the presence of these maximums is not substantiated by the ultrasonic measurements. For example, the weak *b* axes maximums in Figure 1*B* should produce a relatively low longitudinal wave velocity, V_{A13}, which is not observed. This illustrates the advantages of using velocities that provide averages of the degree of orientation for large numbers of crystals over standard optical techniques.

Wave propagation in dunite *B* is similar to wave propagation in orthorhombic crystals. This specimen has strong concentration of all three olivine axes (Figure 1). The fastest longitudinal wave velocity is found for propagation parallel to the olivine *a* axes maximum, whereas propa-

TABLE 4. Elastic Stiffnesses, mb

	Pressure, kb					
	1.0	2.0	4.0	6.0	8.0	10.0
			Dunite A			
c_{11}	2.00	2.02	2.05	2.07	2.08	2.09
c_{33}	2.40	2.43	2.47	2.50	2.52	2.53
c_{44}	0.68	0.69	0.70	0.71	0.71	0.71
c_{12}	0.77	0.79	0.80	0.80	0.81	0.82
c_{13}	0.98	0.98	1.00	1.02	1.02	1.02
			Dunite B			
c_{11}	2.65	2.69	2.74	2.76	2.78	2.80
c_{22}	1.95	1.97	2.00	2.02	2.04	2.05
c_{33}	2.16	2.19	2.23	2.25	2.27	2.29
c_{44}	0.71	0.72	0.74	0.75	0.76	0.76
c_{55}	0.79	0.79	0.80	0.81	0.82	0.82
c_{66}	0.72	0.73	0.75	0.76	0.77	0.78
c_{12}	0.96	0.98	0.98	0.99	1.00	1.02
c_{13}	0.73	0.77	0.79	0.81	0.83	0.85
c_{23}	0.68	0.69	0.70	0.72	0.74	0.76

gation parallel to the *b* axes concentration is characterized by the lowest velocity. This is in agreement with single-crystal measurements of olivine. Two transverse waves with different velocities are propagated parallel to each olivine maximum (Table 3). For this specimen, the tranverse wave velocities are $V_{B2} \simeq V_{B6}$, $V_{B3} \simeq V_{B8}$, and $V_{B5} \simeq V_{B9}$, which confirms the orthorhombic symmetry of the sample.

The velocities for most of the cross checks of the two dunites agree remarkably with single-crystal elastic theory. However, the two dunites do not behave as perfect single crystals. This is not surprising, since several different cores from each dunite were used for the measurements in Table 3. As is shown by their densities (Table 3), the cores differ slightly from one another in accessory mineral content. In addition, olivine orientation most likely varies slightly from core to core.

By using the velocities in Table 3 and mean values of the cross checks, the elastic stiffnesses have been calculated for the two dunites. These are given in Table 4 for several pressures. The mean atomic weights and the Voigt and Reuss averages for isotropic aggregates are given at several pressures for both dunites in Table 5. The formulas and significance of these two averaging techniques have been discussed in many papers [e.g., *Hearmon,* 1961; *Birch,* 1961; *Christensen,* 1965] and will not be repeated here. Using the standard equation for accidental errors in compound quantities [*Topping,* 1966], the VRH averages of the bulk and shear moduli are accurate to 6% and 2%, respectively.

The physical properties for isotropic aggregates of the two dunites are compared in Table 5 with the elastic properties measured for a single crystal of olivine ($Fo_{93}Fa_7$) by *Kumazawa and Anderson* [1969]. The differences in the properties of the dunites and the olivine single crystal are due to many factors that complicate the elasticity of rocks. Both rock samples contain accessory minerals which undoubtedly influence their elastic properties. In addition, the olivine in the dunites is slightly more iron-rich than the single-crystal olivine studied by Kumazawa and Anderson.

Dunite A contains more than 10% serpentine, which has been shown by *Christensen* [1966*b*] to have a significant effect on the elastic properties of dunites and peridotites. The relatively high Poisson's ratio and the lower velocities for dunite A (Table 3) are a consequence of this partial serpentinization. Partial serpentinization also appears to influence the pressure derivatives of the velocities of dunite. *Birch* [1961] has shown that the closure of grain boundary cracks is responsible for the sharp rise in velocity with increasing pressure below 2 kb. Above a few kilobars, the change in velocity with increasing pressure was interpreted as an intrinsic property related to the elasticity of the constituent minerals. The pressure derivatives of the longitudinal wave velocity for dunite B above 2 kb are somewhat higher than the pressure derivatives for single-crystal olivine, whereas the pressure derivatives for the transverse wave velocities are close to single-crystal measurements. This suggests that above 2 kb grain boundary cracks are still influencing the longitudinal velocities but have little effect on the transverse wave velocities. The pressure derivatives for dunite A, on the other hand, are in closer agreement with the olivine single crystal measurements. Serpentinization in dunite A is common along grain boundaries and has most likely eliminated the effect of grain boundary cracks on velocities above a few kilobars.

The elastic properties of rocks are usually described in terms of the theory of isotropic elasticity. In calculating the elastic constants of rocks, it is common to use mean velocities for three propagation directions [e.g., *Birch,* 1961; *Christensen,* 1966*a, b*]. Since the calculations usually involve squared velocities, the resulting accuracies in the elastic constants are considerably less than the accuracies of the velocities. In highly anisotropic rocks such as dunite, mean velocities for measurements in only three orthogonal directions produce additional uncertainties in calculated isotropic constants. Examination of Tables 3 and 5 suggests, however, that simple mean velocities from measurements in three mutually perpendicular directions give values close to true isotropic elastic properties in highly anisotropic rocks. The mean longitudinal velocity at 10 kb from the velocities V_{A1}, V_{A4}, and V_{A7} is 8.24 km/sec, compared to a VRH value of 8.29 km/sec. For dunite B, mean longitudinal-wave velocity at 10 kb determined from the orthogonal set V_{B1}, V_{B4}, and V_{B7} is 8.42 km/sec. This agrees well with the VRH value of 8.45 km/sec.

TABLE 5. Aggregate Properties of Single-Crystal Olivine and Dunite*

Pressure, kb	V_p, km sec^{-1} V	V_p, km sec^{-1} R	V_s, km sec^{-1} V	V_s, km sec^{-1} R	VRH Bulk Modulus, mb	VRH Shear Modulus, mb	VRH Poisson's Ratio	VRH $\partial V_p/\partial P$, km sec^{-1} mb^{-1}	VRH $\partial V_s/\partial P$, km sec^{-1} mb^{-1}	Density, g cm^{-3}
				Olivine [*Kumazawa and Anderson*, 1969], m = 20.79						
10^{-3}	8.48	8.36	4.94	4.84	1.29	0.79	0.25	10.2	3.6	3.311
				Dunite A, m = 20.84						
1.0	8.14	8.09	4.42	4.40	1.30	0.63	0.28			3.268
2.0	8.19	8.14	4.45	4.43	1.32	0.64	0.29	34.2	18.6	3.271
4.0	8.25	8.20	4.48	4.46	1.34	0.65	0.29	23.5	11.1	3.276
6.0	8.29	8.23	4.49	4.48	1.36	0.66	0.29	14.8	5.0	3.281
8.0	8.31	8.25	4.50	4.49	1.37	0.66	0.29	8.0	4.9	3.286
10.0	8.32	8.26	4.51	4.50	1.37	0.67	0.29			3.291
				Dunite B, m = 20.98						
1.0	8.25	8.17	4.71	4.67	1.27	0.73	0.26			3.319
2.0	8.31	8.23	4.73	4.70	1.29	0.74	0.26	40.3	22.7	3.322
4.0	8.38	8.29	4.77	4.74	1.31	0.75	0.26	27.0	16.0	3.327
6.0	8.42	8.34	4.80	4.77	1.33	0.76	0.26	20.4	7.7	3.332
8.0	8.46	8.38	4.81	4.78	1.35	0.77	0.26	17.0	3.0	3.337
10.0	8.49	8.40	4.81	4.78	1.36	0.77	0.26			3.343

* V, Voigt average; R, Reuss average; VRH, Voigt-Reuss-Hill average; m, mean atomic weight.

An additional indication of the error involved in using the mean of three velocities for the calculation of isotropic elastic constants is given by a comparison of the Voigt-Reuss velocities of dunite *B* with measurements from a sample of Twin Sisters dunite of similar density by *Birch* [1960] and *Simmons* [1964]. Birch reported longitudinal wave velocities at 10 kb in three directions of 8.07, 8.23, and 8.95 km/sec. The mean of these measurements, 8.42 km/sec, agrees well with the Voigt and Reuss averages in Table 5. Transverse wave velocities at 10 kb measured by *Simmons* [1964] for the same sample studied by Birch are 4.70, 4.88, and 4.90 km/sec. The mean velocity of 4.83 km/sec for this rock is also in close agreement with the Voigt and Reuss averages for dunite B.

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